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Lateral Stability of Aircraft Nose-Wheel Landing Gear with Closed-Loop Shimmy Damper

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Shimmy of aircraft nose-wheel landing gear is a coupled lateral and torsional dynamic instability that may occur during take off, taxi, and landing. The potential for shimmy is often discovered when the aircraft undergoes taxi (runway) tests. At this stage the options available to the designer are limited, often to introducing a damper in the steering degree of freedom. This paper presents modeling and analysis of shimmy of a 3 degrees-of-freedom linear nose-wheel landing gear model with a closed-loop shimmy damper mechanism attached to its hydraulic steering. A typical control law is considered for representing the closed-loop hydraulic control in the steering rack and pinion system. The stability of the system is studied using root locus plots. The results for critical velocity obtained for different values of gain in the closed-loop shimmy damper show that it can be simulated by an equivalent open-loop shimmy damper with appropriate value of damping. This suggests that an equivalent open-loop shimmy damper model can be used in place of the closed-loop shimmy damper to simplify the modeling of integrated airframe—landing gear dynamics.

Nomenclature

C_L = control law transfer function
C_S = structural lateral damping coefficient
C_{Sh} = equivalent open-loop viscous damping coefficient in torsion
C_Δ = tire lateral damping coefficient
C_θ = equivalent structural damping coefficient in torsion
F_N = side force due to lateral deformation of the tire
G = control gain
I = moment of inertia of wheel-strut assembly about gear vertical axis
K_S = lateral stiffness of the landing gear
K_Δ = lateral stiffness of the tire
K_θ = torsional stiffness of the landing gear
L = distance of the axis of wheel rotation from gear vertical axis
$L_{\mathrm{c.g.}}$ = distance of center of gravity of the wheel-strut assembly from gear vertical axis
m = mass of wheel-strut assembly
p = state vector
s = Laplace variable
V = aircraft forward taxi velocity
V_{Cr} = critical shimmy velocity
x = response vector
y = strut lateral deflection
Δ = tire contact patch centerline lateral deflection with respect to wheel center plane
δ = control input
θ = torsional rotation (yaw) about the strut vertical centerline
σ = modal damping parameter
Ω = ratio of landing gear uncoupled torsional frequency to lateral frequency $= (\omega_\theta/\omega_S)$
ω = modal frequency
ω_S = uncoupled landing gear lateral frequency $= \sqrt{(K_S/m)}$
ω_θ = uncoupled landing gear torsional frequency $= \sqrt{(K_\theta/I)}$

I. Introduction

MATHEMATICAL modeling and analysis of nose-wheel landing gear (NLG) shimmy instability has been studied by a number of authors over the years. Shimmy may be caused by a number of conditions such as low torsional stiffness of the strut, low structural damping, excessive free play in the gear, wheel imbalance, or worn parts. The potential for shimmy of the NLG is often discovered when the aircraft undergoes taxi (runway) tests. At this stage geometrical and stiffness changes may neither be feasible nor weight efficient, limiting the options available to the designer. However, it has been found that there is scope for introducing additional damping into the system by a damper in the steering degree of freedom and it has been found to be effective in the control of shimmy in many situations. But this is achieved at the cost of additional weight and maintenance.

There are studies [1–6] available in open literature on the evaluation of the damping required to control shimmy. Though shimmy dampers have been in use for many years as a measure for controlling shimmy instability, there is no literature in the open domain reporting the study of the effect of shimmy damper as a closed-loop system.

Development of analytical models for the analysis of dynamics of NLG has also been reported by a number of authors [7–11]. Moreland [7] has presented a description of the landing gear system as having strut deflection, swivel angle, tire deflections, linkage strains, and airframe motion as degrees of freedom (DOF). His model accounts for torsional and lateral rigidities of the strut, wheel moment of inertia, and the weight of the strut along with tire elasticity. He has studied landing gear shimmy with basic assumptions involving the part played by the tire, airframe, and the gear, and has discussed the effect of changing gear parameters on the stability of the gear. Howard [8] presented an analytical model to study the effect of geometry of caster system and inertia distribution considering a single wheel configuration. Krabacher [9,10] has presented detailed mathematical models with tire lateral deflection, tire twist, swivel

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angle, strut deflection, wheel tilt as DOF taking into account structural inertias and flexibilities along with some system nonlinearities. Esmailzadeh [11] has developed a dynamical model for aircraft nose gear and has studied the effect of variation of the energy absorption coefficient in the shimmy damper and the location of center of gravity of the landing gear on its transient response. The problem of analyzing a wheeled system for shimmy also requires the knowledge of the tire characteristics and how the forces of contact at the road surface are transmitted to the wheel. Valid mathematical relationships describing the force-deflection characteristics of the tire must be used to predict onset of shimmy correctly. Several theoretical models for tire dynamics have been proposed and used in the literature. Of these, the most commonly used models are the Moreland point contact model [9–12] and the Von Schlippe and Dietrich stretched string model [13–15].

This paper presents modeling and analysis of shimmy of an NLG with a closed-loop shimmy damper (CLSD) mechanism attached to its hydraulic steering. A typical control law transfer function representing the closed-loop hydraulic control in its steering rack and pinion system is used to study closed-loop NLG dynamics. The stability of the system is studied using root locus plots.

II. Analytical Formulation

Figure 1 shows a typical NLG configuration [16] and a schematic representation of the same as a cantilever supported from the fuselage. The landing gear lateral bending causes lateral motion of the wheel in the Y direction. The wheel is free to swivel about the vertical (Z) axis when the steering is not engaged. This degree of freedom is used to steer the aircraft. The steering is incorporated with two spring-loaded hydraulic steering cylinders that serve as a steering mechanism. The steering force generated by the hydraulic actuator is transferred to the wheels using a rack and pinion mechanism through a torque link. The same mechanism is also used to suppress torsional oscillations automatically. The damping is accomplished by the metering of hydraulic fluid through a small orifice between two cylinders. In the present study, the NLG is assumed to be equipped with such a damping mechanism. Schematic of such a steering mechanism is shown in Fig. 2. The NLG model considered for this paper accounts for structural inertia, stiffness, and structural damping. The torsional stiffness of the strut is the result of stiffness offered by torque link, hydraulic spring in the steering actuator, and the structure above torque link including the retraction jack. The damping in the system is assumed to result from the inherent structural damping in lateral and torsional degree of freedom and external viscous damping in the torsional degree of freedom coming from the shimmy damper in the steering mechanism. The landing gear nonlinearities, viz., nonlinear lateral strut flexibility, friction in the oleo strut, free play in the steering, and nonlinear tire behavior are ignored.

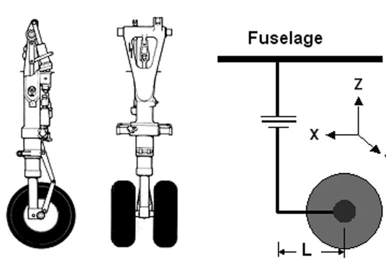

Fig. 1 Schematic of nose-wheel landing gear.

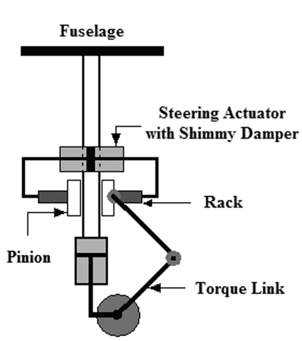

Fig. 2 Details of aircraft steering mechanism with shimmy damper.

Figure 3 shows the schematic of the 3-DOF NLG model having lateral displacement of the wheel (y), angular wheel motion (yaw) about vertical axis (θ) and lateral deflection of the tire contact patch with respect to wheel center plane (Δ) as DOF [2]. It is assumed that the mass of swivel wheel assembly m is lumped at the center of gravity (c.g.) of the wheel assembly. Let F_N be the side force acting on the wheel due to tire lateral deformation Δ. Assuming θ to be small ($\cos\theta = 1$ and $\sin\theta = \theta$) and that there is no tire slippage with respect to ground, the equations of motion [2–4] for the 3-DOF NLG model can be written as

$$m\ddot{y} + mL_{\text{c.g.}}\ddot{\theta} + C_S\dot{y} + K_S y + F_N = 0 \tag{1}$$

$$I\ddot{\theta} + mL_{\text{c.g.}}\ddot{y} + C_\theta\dot{\theta} + K_\theta\theta + F_N L = 0 \tag{2}$$

$$V\theta + \dot{y} + L\dot{\theta} - \dot{\Delta} = 0 \tag{3}$$

Equation (3) represents the kinematic condition that represents zero tire lateral slip with respect to ground. According to the Moreland point contact model, the interaction between the ground and the tire could be treated as occurring at a single point and the elastic restoring effect of the tire on the wheel, that is, side force F_N is assumed to be linearly proportional to the lateral deflection of the tire Δ and to the rate of its change with time $\dot{\Delta}$. Thus, F_N is given by

$$F_N = K_\Delta\Delta + C_\Delta\dot{\Delta} \tag{4}$$

Equations (1–4) can be written in matrix form as

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = 0 \quad \text{where } \{x\} = \begin{bmatrix} y & \theta & \Delta \end{bmatrix}^T \tag{5}$$

The dynamics of the NLG system augmented with control terms can be written as

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} + \{B_\delta\}\delta = 0 \tag{6}$$

where

$$\{B_\delta\} = \{0 \quad K_\theta \quad 0\}^T \tag{6a}$$

and δ is given by

$$\delta = C_L(s)\theta = C_L(s)\{0 \quad 1 \quad 0\}\{x\} \tag{7}$$

In this paper the steering is assumed to be accomplished through the hydraulic rack and pinion-type power steering unit mounted on top of the NLG assembly. The control input in the closed-loop

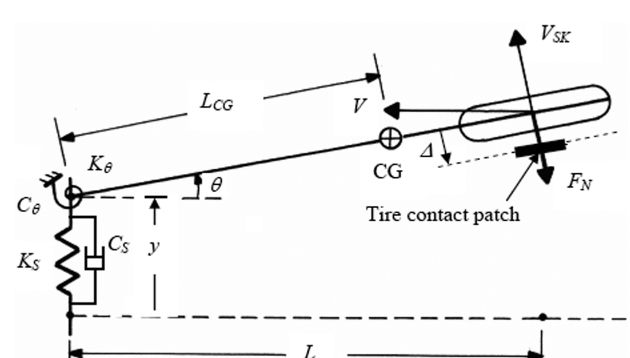

Fig. 3 Simplified nose-wheel landing gear model.

system here is the yaw angle θ and $C_L(s)$ models the transfer function of the electrohydraulic rack and pinion actuation system along with the shimmy damper that closes the loop. The unit is controlled by rudder pedal inputs. The unit also provides nose gear centering when the gear is fully extended. The centering function is accomplished by allowing retract pressure on both sides of the rack, which forces them to the centered position. A centering logic valve, which is located within the steering rack, regulates the differential pressure by venting one side or the other to return until the gear is centered. The nose-wheel steering hydraulic pressure is controlled by the nose-wheel steering shutoff valve. The orifice in the shimmy damper damps the movement of the wheels while the steering system is deactivated, that is, with the nose wheel steering hydraulic power cut off, the nose-wheel will caster and the shimmy damping will be effective. Figure 4 shows a schematic of the rack and pinion hydraulic actuation system with shimmy damper. The transfer function of the electro-hydraulic rack and pinion steering actuation system considered in this paper is given by

$$C_L(s) = \frac{G}{(as^3 + bs^2 + cs + G)} \tag{8}$$

where

$$a = 0.018G, \qquad b = 0.142, \qquad c = (1.128G + 1.131)\sqrt{G} \tag{8a}$$

In Eq. (8) the values of constants a, b, and c are obtained using numerical simulation studies of the steering system model. These simulations are used to validate the model and also to tune the values of the constants a, b, and c by matching the frequency response characteristics of the model with the nose-wheel steering actuator system. Substituting Eq. (8) into Eq. (6) the dynamics of 3-DOF NLG system with closed-loop hydraulic damper can be written in the Laplace domain as

$$(s^2[M] + s[C] + [K] + \{B_\delta\}C_L(s)\{A_1\})\{x\} = 0 \tag{9}$$

The roots of Eq. (9) are calculated for various values of forward velocity, and the velocity corresponding to the critical condition of instability (shimmy) of the closed-loop system is determined. The governing equations of motion representing the previous NLG dynamics model can be written in state space form as

$$\{\dot{p}\} = [A]\{p\} \tag{10}$$

where $[A]$ is a coefficient matrix with real elements and $\{p\}$ is the state vector given by

$$\{p\} = \{y \quad \theta \quad \Delta \quad \dot{y} \quad \dot{\theta}\}^T \tag{11}$$

The previous equations can be transformed in the Laplace domain to yield algebraic equations in terms of the Laplace variable s. The eigenvalues of the matrix $[A]$ yields roots that describe the free vibration modes of the system. In general, the roots of the characteristic equation are complex ($s = \sigma + i\omega$) wherein the real part (σ) is a measure of the modal damping and the imaginary part (ω) is a measure of modal circular frequency. Root locus plots are obtained for various values of forward velocity. The onset of instability is observed as a mode with zero damping at the critical velocity.

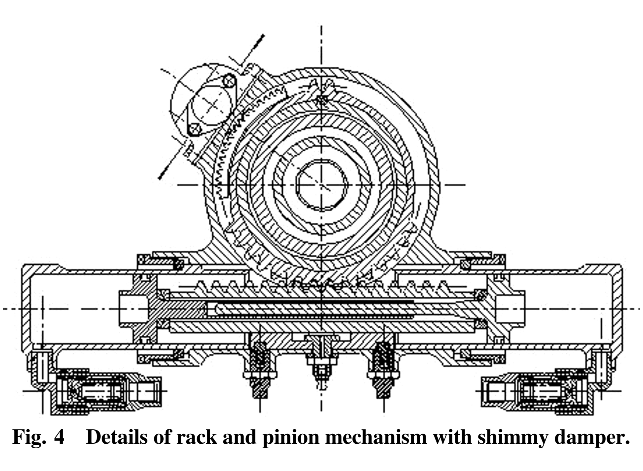

Fig. 4 Details of rack and pinion mechanism with shimmy damper.

III. Results and Discussion

Using the solution method discussed previously, the stability of the NLG system is studied by root locus plots for different values of velocity. Stability studies are carried out on the previous NLG configuration with rack and pinion mechanism provided with the CLSD mechanism. Results are obtained for critical shimmy velocity and shimmy frequency for the range of values of problem parameters given in Table 1. Figures 5 and 6 show the loci of complex eigenvalues for different values of forward velocity for the baseline values of problem parameters given, in Table 1 for the open-loop shimmy damper (OLSD) system with $G = 0$ and for CLSD system with $G = 1$, respectively. The arrows indicate the direction of the movement of the eigenvalues with increase in velocity. The critical condition of shimmy instability is observed where the root locus crosses the imaginary axis. These figures also show the variation of σ (as a measure of modal damping) with forward velocity. The onset of instability is observed as a mode with zero damping ($\sigma = 0$). For the previous cases of open and closed-loop systems the instability occurs at a velocity $V_{\mathrm{Cr}} = 255$ km/h and 280 km/h, respectively. Figures 7 and 8 show variation of critical shimmy velocity and shimmy frequency with the control loop shimmy damper gain G for $\Omega = 3$ and 2, respectively. It can be seen that for the class of NLG configuration considered, using a CLSD and choosing an appropriate gain, an increase of about 15 to 20% in shimmy velocity can be achieved.

To examine whether the effect of the CLSD can be simulated by an equivalent additional damping in the torsional degree of freedom in an open-loop damper, the net damping produced by the shimmy damper is taken into account here as an effective viscous damping in parallel with the overall spring stiffness. Equation (2) is modified to include an additional damping term $C_{\mathrm{Sh}}\,\dot{\theta}$ and is written as

$$I\ddot{\theta} + mL_{\mathrm{c.g.}}\ddot{y} + (C_\theta + C_{\mathrm{Sh}})\dot{\theta} + K_\theta\theta + F_N L = 0 \tag{12}$$

Equation (12), along with Eqs. (1) and (3), is solved for complex eigenvalues to obtain the critical values for shimmy velocity and shimmy frequency. Figure 9 shows variation of shimmy velocity and shimmy frequency with equivalent shimmy damping C_{Sh} in the torsional degree of freedom for base values of NLG parameters and for $\Omega = 2$ and 3 for the OLSD systems. The value of $C_{\mathrm{Sh}} = 100$ $(\mathrm{N} \cdot \mathrm{m})/(\mathrm{rad/s})$ corresponds to approximately 12% of the critical damping in the torsional mode for $\Omega = 3$. It can be seen from Figs. 7–9 that, for the case of $\Omega = 3$, a gain of $G = 1$ in the closed-loop system yields the same value of shimmy velocity (280 km/h) as the open-loop system with the value of 100 $(\mathrm{N} \cdot \mathrm{m})/(\mathrm{rad/s})$ for the damping parameter C_{Sh}. It can also be seen from Figs. 7–9 that the difference in the shimmy frequencies of the CLSD and the corresponding OLSD is about 1%. For the case $\Omega = 3$ the shimmy

Table 1 Values of NLG parameters for baseline problem and their ranges

Baseline values of NLG parameters	
Strut inertia parameters	$m = 22$ kg & $I = 0.198$ kg m^2
Strut geometric parameters	$L = 0.075$ m & $L_{\mathrm{c.g.}} = 0.0675$ m
Strut stiffness parameters	$K_S = 542.83$ kN/m & $\Omega = 3$
Strut damping parameters	$C_S = 0.01\%$; $C_\theta = 0.02\%$
Tire parameters	$K_\Delta = 238.75$ kN/m & $C_\Delta = 205$ Ns/m
Range of values of NLG parameters	
Forward velocity	$V = 0 - 300$ km/h
Strut stiffness	$\Omega = 2$–3
Closed-loop gain	$G = 0$–10

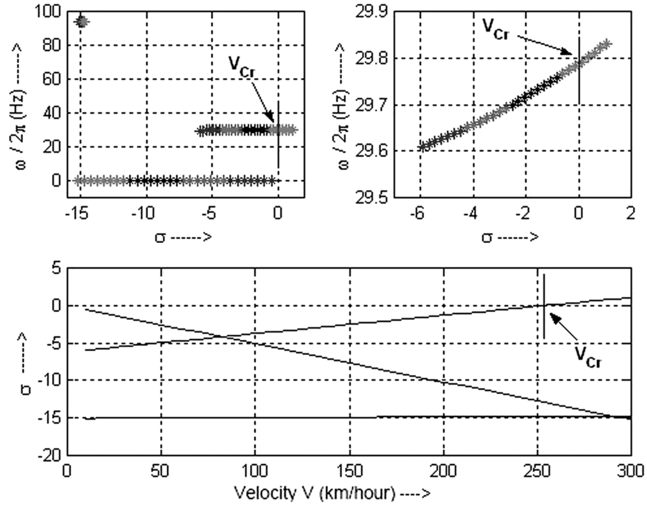

Fig. 5 Root locus and modal damping plots showing critical condition of shimmy for $G = 0$ (open-loop system) for the case $\Omega = 3$: $V_{Cr} = 255$ km/h.

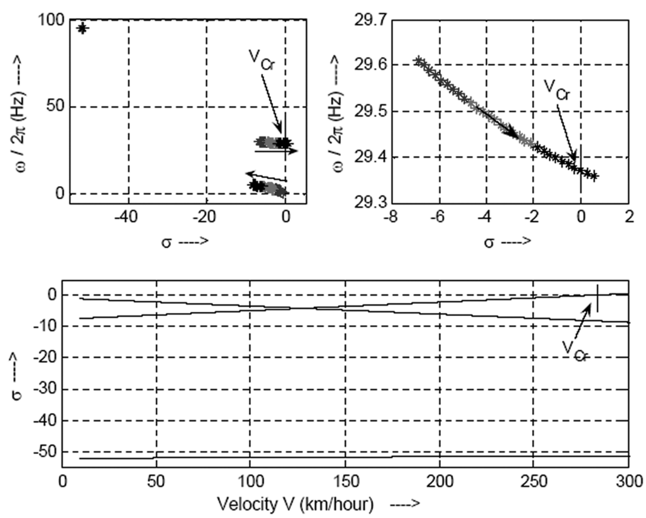

Fig. 6 Root locus and modal damping plots showing critical condition of shimmy for $G = 1$ (closed-loop system) for the case $\Omega = 3$: $V_{Cr} = 280$ km/h.

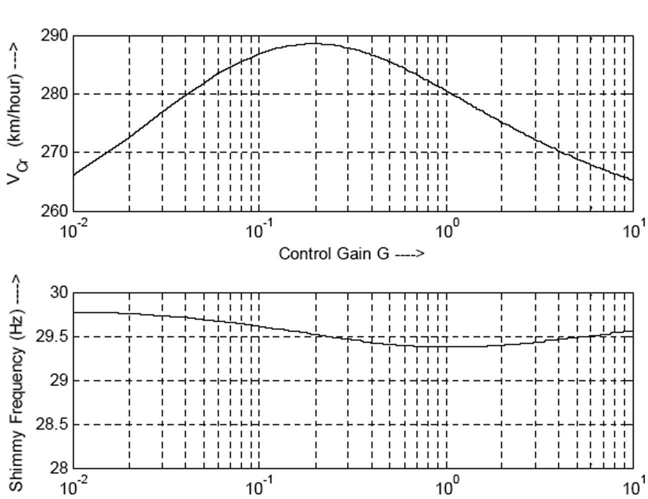

Fig. 7 Variation of V_{Cr} and shimmy frequency with CLSD gain G for the case $\Omega = 3$.

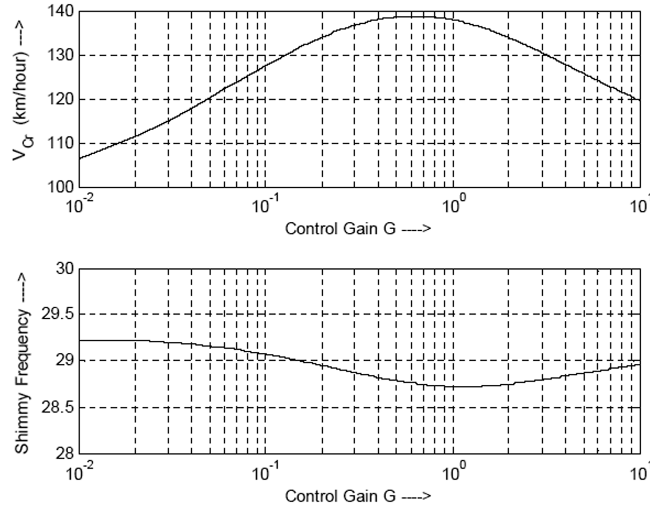

Fig. 8 Variation of V_{Cr} and shimmy frequency with CLSD gain G for the case $\Omega = 2$.

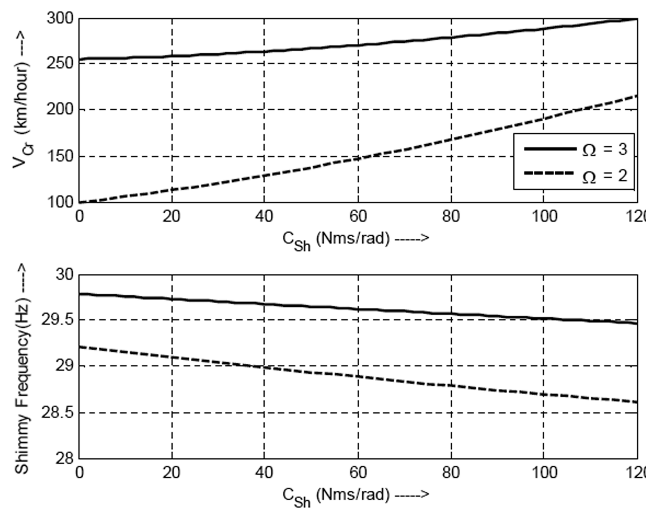

Fig. 9 Variation of V_{Cr} and shimmy frequency with OLSD damping parameter C_{Sh} for $\Omega = 2$ and 3.

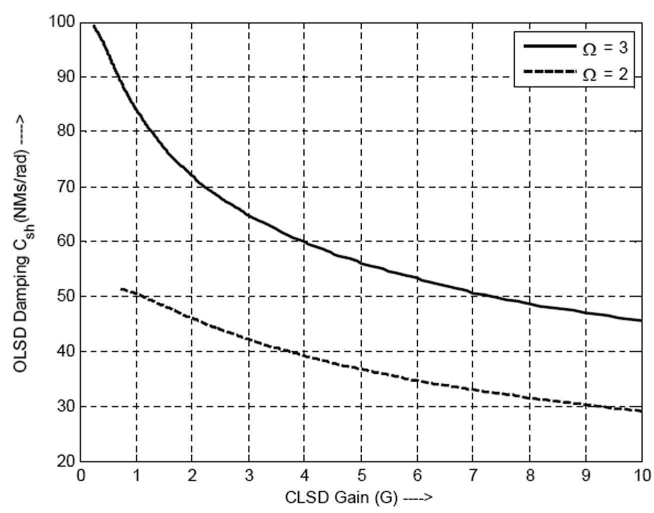

Fig. 10 Correspondence between the CLSD gain G values with that of the OLSD damping parameter C_{Sh} to yield identical shimmy velocities for $\Omega = 2$ and 3.

frequency values for CLSD and corresponding OLSD systems are 29.36 and 29.78 Hz, respectively. Figure 10 shows the correspondence between the gain values of the CLSD with that of the damping parameter C_{Sh} of the OLSD that can yield identical shimmy velocities.

IV. Conclusions

The study of shimmy stability of simplified 3-DOF nose-wheel landing gear model with closed-loop shimmy damper dynamics was presented. System stability was examined using root locus plots and the effect of closed-loop shimmy damper dynamics on the onset of shimmy was studied. It was observed that a significant increase in system stability can be achieved by choosing appropriate gain in the closed-loop shimmy damper. The results for critical velocity obtained for different values of gain show that the effect of the closed-loop shimmy damper can be simulated by an equivalent open-loop shimmy damper with the appropriate value of damping. This suggests that an equivalent open-loop shimmy damper model can be used in place of the closed-loop shimmy damper to simplify the modeling effort which is particularly of value in the modeling and analysis of the dynamics and stability of nose-wheel landing gear integrated with overall airframe dynamics.

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